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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

335. Proposed by **PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.**

A person has $1800 in notes payable $18 monthly, bearing 10% interest. Find their present value if the interest is payable at the maturity of each note; also present value if interest is payable annually. [An actual business transaction.]

Solution by **G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.**

From the reading of the problem, it is supposed that the $1800 is the actual amount to be paid, including interest.

I. Let x=present value.

$$\text{Then } x=\frac{18}{1.00\frac{5}{6}}+\frac{18}{(1.00\frac{5}{6})^2}+\frac{18}{(1.00\frac{5}{6})^3}+\ldots+\frac{18}{(1.00\frac{5}{6})^{100}}$$

$$=\frac{18}{(1.00\frac{5}{6})^{100}}[1+1.00\tfrac{5}{6}+(1.00\tfrac{5}{6})^2+\ldots(1.00\tfrac{5}{6})^{99}]$$

$$=\frac{18}{(1.00\frac{5}{6})^{100}}\left(\frac{(1.00\frac{5}{6})^{100}-1}{.00\frac{5}{6}}\right)=2160\left(1-\frac{1}{(1.00\frac{5}{6})^{100}}\right)$$

$$=2160(1-.436114)=1217.994, \text{ the number of dollars.}$$

II. In such transactions the interest for each year is calculated as though no payments were made. Hence,

$$x(1.10)^8(1.03\tfrac{1}{3})-216[(1.10)^7+(1.10)^6+\ldots+1](1.03\tfrac{1}{3})-72=0.$$

$$x(1.10)^8(1.03\tfrac{1}{3})=2160[(1.10)^8-1](1.03\tfrac{1}{3})+72.$$

$$x=2160\left(1-\frac{1}{(1.10)^8}\right)+\frac{72}{(1.10)^8(1.03\frac{1}{3})}.$$

$$x=\$1184.846.$$

336. Proposed by **V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.**

Evaluate the determinant

$$\Delta=\begin{vmatrix} a_1^2, & a_2^2, & a_3^2, & \ldots & a_n^2 \\ a_2^2, & a_3^2, & a_4^2, & \ldots & a_{n+1}^2 \\ a_3^2, & a_4^2, & a_5^2, & \ldots & a_{n+2}^2 \\ \cdot & \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot \\ a_n^2, & a_{n+1}^2, & a_{n+2}^2, & \ldots & a_{2n-1}^2 \end{vmatrix}$$

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Subtract each row from the one which follows it, beginning with the last but one. Repeat the same operation, stopping at the second row. Keep repeating this operation, leaving out a row each time, until all the rows have been thus omitted; then if D=value of determinant and

$$\triangle^r a_s{}^2 = \triangle^{r-1} a^2{}_{s+1} - \triangle^{r-1} a_s{}^2,$$

we get

$$D = \begin{vmatrix} a_1^2, & a_2^2, & a_3^2, & \dots & a_n^2 \\ \triangle a_1^2, & \triangle a_2^2, & \triangle a_3^2, & \dots & \triangle a_n^2 \\ \triangle^2 a_1^2, & \triangle^2 a_2^2, & \triangle^2 a_3^2, & \dots & \triangle^2 a_n^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \triangle^{n-1} a_1^2, & \triangle^{n-1} a_2^2, & \triangle^{n-1} a_3^2, & \dots & \triangle^{n-1} a_n^2 \end{vmatrix}$$

Repeating the same series of operations on the columns, we get

$$D = \begin{vmatrix} a_1^2, & \triangle a_1^2, & \triangle^2 a_1^2, & \dots & \triangle^{n-1} a_1^2 \\ \triangle a_1^2, & \triangle^2 a_1^2, & \triangle^3 a_1^2, & \dots & \triangle^n a_1^2 \\ \triangle^2 a_1^2, & \triangle^3 a_1^2, & \triangle^4 a_1^2, & \dots & \triangle^{n+1} a_1^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \triangle^{n-1} a_1^2, & \triangle^n a_1^2, & \triangle^{n+1} a_1^2, & \dots & \triangle^{2n-2} a_1^2 \end{vmatrix}$$

If a_r^2 is a function of r of the pth degree in r, whose highest term has a coefficient unity, the quantities a_1^2, a_2^2, a_3^2, ... form an arithmetic series of the pth order.

If $p=n-1$ all the elements below the second diagonal vanish, while all those in it are equal to $(n-1)!$, and $D=(-1)^{n(n-1)/2}[(n-1)!]^n$.

If $m<(n-1)$, $D=0$.

These determinants have been called orthosymmetrical.

GEOMETRY.

361. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

ABCD is a quadrilateral. The bisectors of A and C meet in O_1; those of B and D meet in O_2. Find the tangent of the angle between AD and O_1O_2 in terms of sines and cosines of A, D, $A+B$, and $A+D$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $ABCD$ be the quadrilateral. Produce AB, DC, and AD, BC until they intersect in E, F, respectively. Take ADE as the triangle of reference for trilinear coordinates.

Let $\gamma=0$, be the epuation to AD; $\beta=0$, the equation to AB; $\alpha=0$, the equation to DC; $l\alpha+m\beta+n\gamma=0$, the equation to BC.

Also let $P=\sqrt{[l^2+m^2+n^2-2mn\cos A-2nl\cos D-2ml\cos E]}$.